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STEADY-STATE CONDITIONS OF HIGH-TEMPERATURE PLASMA. A PINCH IN A LONGITUDINAL MAGNETIC FIELD

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Steady-state Conditions of High-temperature Plasma. A Pinch in a Longitudinal Magnetic Field.

E. F. Tkalich and V. S. Tkalich

Steady-state conditions of high-temperature plasma are studied in a kinetic approximation. We investigate the radial distribution of density, azimuthal current, and other macroscopic physical values in a pinch with longitudinal magnetic field.

A pinch compressed by the magnetic field of its own current was analyzed on the basis of a kinetic approximation by Bennett [1] and later by Chyulli and Miku [2]. The investigation of a plasma cylinder in equilibrium by an outside magnetic field was conducted by Tonks [3] who studied in detail the possible forms of electron trajectories. Certain steady-state kinetic problems were examined by Morozov and Solov'ev [4].

Vlasov [5] formulated the general problem of space-bounded plasma; the concept of a plasmoid condition is introduced, and a series of plasmoids are examined with the aid of a distribution function, the exponential curve index of which is quadratic with respect to velocity. H. Grad [6] examined the boundary between a noncolliding plasma and a magnetic field in a Cartesian coordinate system, and he studied

trajectories by means of exact solutions. One of the authors [7] studied the steady-state problem of a high-temperature plasma with the aid of distribution functions which were a generalization of the Maxwellian distribution law. In this article we will investigate the steady-state conditions based on distribution functions closely related to the Maxwellian structure and we will study the fundamental macroscopic characteristics of a pinch in a longitudinal magnetic field.

1. High-temperature plasma can be described by kinetic equations without terms which take into account interaction at small distances [8-11]. Removing Poisson brackets [9, 10, 12] in the kinetic equation for type k ions in an arbitrary orthogonal system of coordinates [13] (q_1, q_2, q_3), we will obtain

$$\frac{\partial f_k}{\partial t} + \sum \frac{\partial(\mathcal{H}_k, f_k)}{\partial(P_{k_\alpha}, q_\alpha)} = 0 \qquad (k = 1, 2, \ldots N).$$

Here N is the number of particle types. Summation with respect to α is performed everywhere from 1 to 3 (α = 1, 2, 3). Canonical variables of charged particles in an electromagnetic field - the Hamiltonian $\mathcal{H}_k$ and components of total momentum P_{k_α} have the form

$$\mathcal{H}_k = \sum \frac{p_{k_\alpha}^2}{2m_k h_\alpha^2} + e_k \varphi, \quad P_{k_\alpha} = p_{k_\alpha} + \frac{e_k}{c} h_\alpha A_\alpha, \quad p_{k_\alpha} = m_k h_\alpha^2 W_{k_\alpha}.$$

The values h_α, p_{k_α}, W_{k_α} are the Lamé coefficient, a component of the momentum of type k ions, and generalized velocity, respectively; φ and A are electromagnetic potentials.

Temperature is presented as a dispersion of linear velocity [14]; thus, for temperature T_{k_α} of type k ions in direction α we will have the following expression:

$$h_s^2 m_k \varkappa T_{k_s} = \overline{(p_{k_s} - \bar{p}_{k_s})^2}. \qquad (1)$$

In the future we will call such a problem in which the basic set of physical values – electrical field, magnetic field, distribution function, and Lame's coefficients- do not depend on time, a steady-state problem. We will call a cyclical coordinate one on which the values of the fundamental composite do not depend.

2. We will examine the steady-state problem in which the coordinate q_3 is cyclical. In this case the function of the integrals of symmetry $f_k = f_k(\mathcal{H}_k, P_{k_3})$ is the partial solution of the kinetic equation. Let the vector potential have only the third component depending on the first two coordinates $\mathbf{A} = A(q_1, q_2)\mathbf{e}_3$, the electrical field does not have the third component. Then from Maxwell's equations we obtain [15] the following system for the determination of φ and $\mathbf{A}$:

$$h_1 h_2 h_3 \Delta \varphi + 4\pi \sum e_l \int f_l dp_1 dp_2 dp_3 = 0,$$

$$h_1 h_2 h_3^3 \Delta^* h_3 A + \frac{4\pi}{c} \sum \frac{e_l}{m_l} \int f_l p_3 dp_1 dp_2 dp_3 = 0,$$

$$\Delta^* = \frac{1}{h_1 h_2 h_3} \left(\frac{\partial h_2 \partial}{\partial q_1 h_1 h_3 \partial q_1} + \frac{\partial h_1 \partial}{\partial q_2 h_2 h_3 \partial q_2} \right).$$

Summation with respect to l is performed everywhere from 1 to N(l = = 1, 2,..., N), Δ is the Laplace operator [13].

Let the exponent of the distribution function be a linear Hamiltonian function $\mathcal{H}_k$ and of momentum P_{k_3}. Then for the distribution function and density n_k we will obtain the following expressions

$$f_k = \frac{n_k}{(2\pi m_k \varkappa T_k)^{3/2}} \exp\left(-\frac{1}{2m_k \varkappa T_k}\right) \left[\left(\frac{p_{k1}}{h_1}\right)^2 + \left(\frac{p_{k2}}{h_2}\right)^2 + \left(\frac{p_{k3} - m_k h_3^2 w_k}{h_3}\right)^2\right]$$

$$n_k = n_{k0} \exp\left(-\frac{1}{2m_k \varkappa T_k}\right) \left[2m_k e_k \varphi - 2\frac{e_k}{c} m_k w_k h_3 A - (m_k w_k h_3)^2\right].$$

In the future, the values n_{k_0} and w_{k_α} will everywhere designate constant dimensions of density and of generalized velocity respectively; T_k is constant in the sense of temperature. With the aid of the distribution function we will determine the third component of the ordered velocity V_k.

$$V_k = h_3 w_k. \tag{2}$$

If the exponent of the distribution function is a linear Hamiltonian function $\mathcal{H}_k$ and quadratic function of momentum P_{k_3}, then f_k and n_k have the form

$$f_k = \frac{n_k \sqrt{1 + a_k h_3^2}}{(2\pi m_k \varkappa T_k)^{3/2}} \exp\left(-\frac{1}{2m_k \varkappa T_k}\right)\left\{\left(\frac{p_{k1}}{h_1}\right)^2 + \left(\frac{p_{k2}}{h_2}\right)^2 + \right.$$

$$\left. + (1 + a_k h_3^2)\left[\frac{p_{k3}}{h_3} + \frac{b_k h_3 + 2a_k h_3^2 \frac{e_k}{c} A}{2(1 + a_k h_3^2)}\right]^2\right\},$$

$$n_k = \frac{n_{k0}}{\sqrt{1 + a_k h_3^2}} \exp\left(-\frac{1}{2m_k \varkappa T_k}\right)\left[2m_k e_k \varphi + \left(\frac{e_k}{c} A\right)^2 - \right.$$

$$\left. - \left(\frac{e_k}{c} A - \frac{b_k h_3}{2}\right)^2 / (1 + a_k h_3^2)\right].$$

Here a_k and b_k are constants. The value $1 + a_k h_3^2$ is essentially positive in the region where there is plasma. At the points where A becomes infinity when $a_k > 0$ there will be no plasma; when $a_k < 0$ the plasma accumulates at these points. With the aid of the aforementioned distribution function and Relation (1) we will find the third component of velocity V_k and temperature T_{k_3} in a selected direction

$$V_k = -\frac{b_k h_3 + 2a_k h_3^2 \frac{e_k}{c} A}{2m_k (1 + a_k h_3^2)}, \quad T_{k3} = \frac{T_k}{1 + a_k h_3^2}. \tag{3}$$

3. We will examine the steady-state problem with two cyclical coordinates q_2 and q_3. The arbitrary function of symmetry integrals $f_k = f_k(\mathcal{H}_k, P_{k_2}, P_{k_3})$ is a partial solution of the kinetic equation.

If $\mathbf{A}$ and φ depend only on the first coordinate, then assuming $A_1 = 0$, we will obtain from Maxwell's equations [15] the following system of equations for determining φ, A_2, and A_3:

$$\left.\begin{aligned} \frac{d}{dr} r^{\sigma} \frac{d\varphi}{dr} + 4\pi \sum e_e \int f_e \, dp_1 dp_2 dp_3 &= 0, \\ r^{2\sigma} \frac{d}{dr} \frac{1}{r^{\sigma}} \frac{dr^{\sigma} A_2}{dr} + \frac{4\pi}{c} \sum \frac{e_e}{m_e} \int f_e p_2 \, dp_1 dp_2 dp_3 &= 0, \\ \frac{d}{dr} r^{\sigma} \frac{dA_3}{dr} + \frac{4\pi}{c} \sum \frac{e_e}{m_e} \int f_e p_3 \, dp_1 dp_2 dp_3 &= 0. \end{aligned}\right\} \tag{4}$$

The constant σ takes the values 0 and 1 in a Cartesian and cylindrical system of coordinates respectively; $\underline{r}$ is either a Cartesian coordinate or a distance from the axis of symmetry.

Let the exponent of the distribution function be a linear function of the Hamiltonian and of momenta. Then for the distribution function and density we will obtain the following expressions:

$$f_k = \frac{n_k}{(2\pi m_k \varkappa T_k)^{3/2}} \exp\left(-\frac{1}{2m_k \varkappa T_k}\right)\left\{p_{k1}^2 + \left(\frac{p_{k2}}{r^{\sigma}} - m w_{k2} r^{\sigma}\right)^2 + \left(p_{k3} - m w_{k3}\right)^2\right\},$$

$$n_k = n_{k0} e^{-\psi_k}, \qquad \psi_k = \frac{e_k}{\varkappa T_k}\left(\varphi - \frac{w_{k2}}{c} r^{\sigma} A_2 - \frac{w_{k3}}{c} A_3 - \frac{m_k w_{k2}^2 \sigma r^2}{2e_k}\right).$$

With the aid of the distribution function we will find the second and third components of the ordered velocity

$$V_{k2} = w_{k2} r^{\sigma}, \quad V_{k3} = w_{k3}. \tag{5}$$

We will examine a case where only electrons move, the ions will be considered as stationary and cold. The ions play the role of the background of density n_+, which compensates the space charge of electrons. We will introduce the constants σ_2 and σ_3, in the following manner. In a Cartesian coordinate system $\sigma_2 = \sigma_3 = 0$. In a cylindrical system $\sigma_2 = 0$, $\sigma_3 = 1$ in the case of an azimuthal magnetic field ($A_2 = 0$); and $\sigma_2 = 1$, $\sigma_3 = 0$ in the case of a longitudinal magnetic field ($A_3 = 0$). Substituting the distribution function in (4) for

the determination of φ, A_2, and A_3 we will obtain the following system of equations:

$$\left.\begin{aligned} &\frac{1}{r^2}\frac{d}{dr}\frac{1}{r^2}\frac{d}{dr}(r^2A_2)+\frac{4\pi}{c}ew_2n=0,\\ &\frac{1}{r^3}\frac{d}{dr}r^3\frac{dA_3}{dr}+\frac{4\pi}{c}ew_3n=0,\\ &n_{+}=n+\frac{1}{4\pi e}\frac{d}{dr}r^2\frac{d\varphi}{dr}.\end{aligned}\right\} \qquad (6)$$

A partial solution of this system has the form

$$A_a=S\nu_a\frac{c\varkappa T}{r_0^{\sigma_a}ew_a}\rho^{-\frac{\sigma_2}{2}}\left(\xi+\frac{a}{2}\rho^2+b\rho\right),$$

$$\varphi=\frac{\varkappa T}{e}\left[(1+S\nu_a)\xi+S\nu_a\left(\frac{a}{2}\rho^2+b\rho\right)+\left(\frac{mw_a r_0^2}{2\varkappa T}\sigma_2+2\sigma_3\right)\rho\right],$$

$$\rho=(1-\sigma)\frac{r}{r_0}+\sigma_2\left(\frac{r}{r_0}\right)^2+\sigma_3\ln\left(\frac{r}{r_0}\right),$$

$$\nu_a=\frac{\pi e^2n_0w_a^2r_0^{2\sigma_a+2}}{2^{2(\sigma_a-1)}c^2\varkappa T},$$

$$\rho=C_2\pm\int\frac{d\xi}{\sqrt{2Se^{-\xi}-2a\xi+C_1}}.$$

Here r_0 is a constant of linear dimension; ρ is a dimensionless coordinate; a and b are dimensionless arbitrary constants; S takes two values ($S = \pm 1$). In the case of a longitudinal magnetic field $A_3 = 0$ when $S = 1$, $a > 0$, we will obtain the following class of solutions in which ξ is finite at a finite ρ:

$$A=-\frac{\nu c\varkappa T}{r_0 e w}\rho^{-\frac{1}{2}}(\xi+b\rho),\qquad \varphi=\frac{\varkappa T}{e}\left[(1-\nu)\xi-\nu b\rho+\frac{mw^2r_0^2}{2\varkappa T}\rho\right],$$

$$C_1e^{\xi}=2\operatorname{ch}^2\frac{\sqrt{C_1}}{2}(\rho-C_2);\quad n=\frac{1}{2}C_1n_0\operatorname{ch}^{-2}\frac{\sqrt{C_1}}{2}(\rho-C_2);\quad \nu=\frac{\pi e^2n_0w^2r_0^4}{c^2\varkappa T}.$$

Let the electrical field equal zero. Without lessening the generality, the direction of electron rotation can be considered positive (sign e = −sign w). Then for the magnetic field and density we will obtain the relations

$$\left.\begin{aligned} &H=H_*+(H_\infty-H_*)\frac{H_1-H_*+(H_0-H_*)\operatorname{th}\rho}{H_\infty-H_*+(H_0-H_*)\operatorname{th}\rho},\\ &n=\frac{n_0}{\operatorname{ch}^2(\rho-\rho_0)};\quad \rho=\frac{r^2w}{\sqrt{2C_1}2cR_0};\quad \operatorname{th}\rho_0=\frac{H_0-H_*}{H_*-H_\infty},\\ &R_0^2=\frac{\varkappa T}{4\pi n_0e^2};\quad H_*=-\frac{mwc}{e};\quad H_\infty=-\frac{mwc}{e}-\frac{\varkappa T}{\sqrt{2}eR_0}.\end{aligned}\right\} \qquad (7)$$

Here R_0 is the Debye radius; n_* is the maximum density (at the point $\rho = \rho_*$); H_0, H_∞, H_* are the values of the magnetic field at the axis of symmetry, at infinity, and at the point of maximum density respectively. Here the values w, H_*, H_∞ are always positive and do not vanish; the magnetic field satisfies the inequality

$$\left|\frac{H - H_*}{H_\infty - H_*}\right| < 1.$$

Henceforth, in place of the magnetic field we will examine the Larmor frequency of electrons ($\omega = -\frac{eH}{mc}$). Analysis of Relation (7) shows that the magnetic field is a monotonically increasing function of the radius. In connection with this the Larmor frequency and angular velocity w vary within the following limits

$$\omega_0 < \omega < \omega_\infty, \quad 0 < w < \omega_\infty.$$

If the conditions $w < \omega_0 < \omega_\infty$ are fulfilled, then the density monotonically increases from the column axis, vanishing at infinity. In the case where $\omega_0 = w$ the formulas in (7) take a simple form

$$H = H_0 + (H_\infty - H_0)\,\mathrm{th}\,\rho, \quad n = n_*\,\mathrm{ch}^{-2}\rho.$$

If the conditions $\omega_0 < w < \omega_\infty$ (i.e. $\rho_* > 0$), then the density is maximum when $\rho = \rho_*$, and therefore decreases, vanishing at infinity. In addition, the angular velocity satisfies inequality $\omega_0 + \omega_\infty > 2w$. If the average radius of the column is much larger than the thickness of the plasma layer ($R \gg L$), then we have the relation

$$2w \approx \omega_0 + \omega_\infty.$$

If the angular velocity changes within the limits of $0 < 2w < \omega_\infty$, then the magnetic field changes sign. In this case it can take significantly larger values with respect to the modulus.

Expressing the linear density "effective area" and the maximum density by n_*, we will obtain the relation

$$S = \frac{\pi\sqrt{2C}\,2cR_0}{w}\,\frac{H_\infty - H_0}{H_\infty - H_*}, \quad Sn_* = \int_0^\infty n\,2\pi r\,dr.$$

Assuming $S = \pi L^2$, in the case $w = \omega_0$ we will obtain

$$L^2 = \frac{\sqrt{2C_1}\,2cR_0}{\varpi}.$$

In the case $S = 2\pi RL$ and $w = \frac{(\omega_0 + \omega_\infty)}{2}$ $(R \gg L)$ we will obtain

$$RL = \frac{\sqrt{2C_1}\,2cR_0}{\varpi}.$$

4. In a Cartesian system of coordinates we will examine the distribution in which the exponent is a linear function of Hamiltonian $\mathcal{H}_k$ and a quadratic function of momenta P_{k_2}, and P_{k_3}. After identical transformation, the distribution function and density take the form

$$f_k = n_k F_k(p_1, p_2, p_3, r),$$

$$n_k = n_{k0} \exp\left(-\frac{1}{2m_k \varkappa T_k}\right)\left\{2m_k e_k(\varphi - \varphi_0) + \left(\frac{e_k}{c}\right)^2 (\mathbf{A}^2 - \mathbf{A}_0^2) - \right.$$
$$\left. - \left(\frac{e_k}{c}\right)^2 (\mathbf{A} - \mathbf{A}_0)^2 \frac{(\mathbf{a}_k^2 + \mathbf{b}_k^2)}{(\mathbf{a}_k \times \mathbf{b}_k)^2}\right\}.$$

$$F_k = \frac{|\mathbf{a}_k \times \mathbf{b}_k|}{(2\pi m_k \varkappa T_k)^{3/2}} \exp\left(-\frac{1}{2m_k \varkappa T_k}\right)\left\{p_{k1}^2 + \left[\mathbf{a}_k\left(\mathbf{p}_k + \frac{e_k}{c}\mathbf{A}\right) + \right.\right.$$
$$\left.\left. + \frac{e_k}{c}(\mathbf{A} - \mathbf{A}_0)\frac{(\mathbf{a} \times \mathbf{b}) \times \mathbf{b}}{(\mathbf{a} \times \mathbf{b})^2}\right]^2 + \left[\mathbf{b}_k\left(\mathbf{p}_k + \frac{e_k}{c}\mathbf{A}\right) + \frac{e_k}{c}(\mathbf{A} - \mathbf{A}_0)\frac{(\mathbf{b} \times \mathbf{a}) \times \mathbf{a}}{(\mathbf{a} \times \mathbf{b})^2}\right]^2\right\}.$$

The vectors $\mathbf{a} \equiv (0, a_2, a_3)$ and $\mathbf{b} \equiv (0, b_2, b_3)$ are constants. The ordered velocity $\mathbf{V}_k \equiv (0, V_{k_2}, V_{k_3})$ is determined from the relations

$$\left.\begin{aligned} -\frac{cm_k}{e_k}(\mathbf{a}_k \mathbf{V}_k) &= (\mathbf{a}_k \mathbf{A}) + \frac{(\mathbf{A} - \mathbf{A}_0)(\mathbf{a} \times \mathbf{b}) \times \mathbf{b}}{(\mathbf{a} \times \mathbf{b})^2}, \\ -\frac{cm_k}{e_k}(\mathbf{b}_k \mathbf{V}_k) &= (\mathbf{b}_k \mathbf{A}) + \frac{(\mathbf{A} - \mathbf{A}_0)(\mathbf{b} \times \mathbf{a}) \times \mathbf{a}}{(\mathbf{a} \times \mathbf{b})^2}. \end{aligned}\right\} \qquad (8)$$

With the aid of Relation (1) we will determine the temperatures in the directions $\mathbf{a}$ and $\mathbf{b}$

$$T_{ka} = \frac{1}{a_k^2} T_k, \quad T_{kb} = \frac{1}{b_k^2} T_k. \qquad (9)$$

5. We will examine a steady-state problem in a cylindrical system of coordinates with two cylindrical coordinates θ and Z. We will determine the values φ, A_θ, and A_Z from the system of (4), assuming $\sigma = 1$.

Let the magnetic field be longitudinal and the exponent of the distribution function be linearly dependent on the Hamiltonian ($\mathcal{H}_k$), and quadratically dependent on the azimuthal component of the total momentum. If a uniform rotation of type k ions does not exist as a whole, and on the column axis there are no peculiarities ($\lim_{r \to 0} \varphi = 0$, $\lim_{r \to 0} rA_\theta = 0$), then dropping the index θ from the azimuthal component of the vector potential, we will write the distribution function and the density as

$$f_k = \frac{n_k(r) F_k(p_{kz}, M_k, p_k, r)}{(2\pi m_k)^{3/2} T_k \sqrt{T_{kz}}},$$

$$n_k(r) = \frac{n_{k0}}{\sqrt{1+\left(\frac{r}{r_{k0}}\right)^2}} \exp\left(-\frac{1}{2m_k T_k}\right)\left[2e_k m_k \varphi + \frac{\left(\frac{e_k r A}{c r_{k0}}\right)^2}{1+\left(\frac{r}{r_{k0}}\right)^2}\right], \tag{10}$$

$$F_k(p_{kz}, M_k, p_k, r) = \sqrt{1+\left(\frac{r}{r_{k0}}\right)^2} \exp\left(-\frac{1}{2m_k T_k}\right) \times$$

$$\times \left\{p_{kz}^2 + \frac{T_k}{T_{kz}} p_k^2 + r^{-2}\left[1+\left(\frac{r}{r_{k0}}\right)^2\right]\left[M_k + \frac{e_k A}{c} \frac{\left(\frac{r}{r_{k0}}\right)^2}{1+\left(\frac{r}{r_{k0}}\right)^2}\right]^2\right\}.$$

In order that the plasma density at infinity be equal to zero and the magnetic field be different from zero, we will consider the constant of integration essentially positive and we will designate it by r_{k0}^2. With the aid of the distribution function we will determine the average value of the azimuthal component of velocity (V_k) of type k ions

$$V_k = -\frac{e_k A}{c m_k} \frac{\left(\frac{r}{r_{k0}}\right)^2}{1+\left(\frac{r}{r_{k0}}\right)^2}. \tag{11}$$

With the aid of Relation (1) we will determine the azimuthal temperature $T_{k\theta}(r)$

$$T_{k\theta}(r)=\frac{T_k}{1+\left(\frac{r}{r_{k0}}\right)^2}. \tag{12}$$

6. The solution of the system of nonlinear equations is associated with substantial mathematical difficulties. However, in the case of a sufficiently small density ($n_{k0}\to 0$) it is permissible to disregard the space charges and currents. Then the solution of system (4) can be selected as

$$A=H\frac{r}{2},\quad \varphi=0, \tag{13}$$

where H is the constant magnetic field strength. Such a selection corresponds to the case where the density at the axis of symmetry is maximum. Substituting (13) into (10) we will find the density distribution

$$n_k(r)=\frac{n_{0k}}{\left[1+\left(\frac{r}{r_{0k}}\right)^2\right]^{3/2}}\exp\left(-\frac{1}{2\varkappa m_k T_k}\right)\frac{\left(\frac{e_k H r^2}{2c r_{0k}}\right)^2}{1+\left(\frac{r}{r_{0k}}\right)^2}. \tag{14}$$

With the determination of macroscopic values we will calculate integrals of type

$$I_{ab}(\epsilon_k)\equiv\int_0^\infty\frac{x^a}{(1+x^2)^b}e^{-\epsilon_k\frac{x^4}{1+x^2}}dx,\quad \epsilon_k\equiv\frac{1}{2\varkappa m_k T_k}\left(\frac{e_k r_{0k} H}{2c}\right)^2 \tag{15}$$

approximately for two limiting cases $\varepsilon_k \gg 1$ and $\varepsilon_k \gg 1$. We will designate by Q_k the number of particles of type k entering into a unit of column length and we will introduce into the examination a "minimal" thickness of ion layer L_k, a Larmor radius of ions R_k^*, and a transverse heat velocity U_k.

$$\pi L_k^2 \equiv \frac{Q_k}{n_{0k}}, \quad R_k^* \equiv \frac{U_k}{|\omega_k|}, \quad \omega_k \equiv \frac{e_k H}{m_k c}, \quad U_k \equiv \sqrt{\frac{\varkappa T_k}{m_k}}. \tag{16}$$

We will determine with the aid of (14)-(16) the constants of integration r_{k_0}

$$r_{k0} = \frac{1}{\sqrt{2\pi}} \frac{L_k^2}{R_k^*} \ (\text{when } \varepsilon_k \gg 1); \quad r_{k0} = \frac{1}{2\sqrt{2\pi}} \frac{L_k^2}{R_k^*} \ (\text{when } \varepsilon_k \ll 1). \tag{17}$$

Eliminating r_{k_0} from (15) we will find the expression for ε_k

$$\varepsilon_k = \frac{1}{\pi}\left(\frac{L_k}{2R_k^*}\right)^4 (\text{when } L_k \gg R_k^*), \quad \varepsilon_k = \frac{1}{\pi}\left(\frac{L_k}{2\sqrt{2}R_k^*}\right)^4 (\text{when } L_k \ll R_k^*). \tag{18}$$

The case $\varepsilon_k \gg 1$ (i.e. $L_k \gg R_k^*$) corresponds to a strong magnetic field and to small heat velocities; the case $\varepsilon_k \ll 1$ (i.e. $L_k \ll R_k^*$) is realized for weak magnetic fields and large heat velocities. Using relations (12), (14)-(18), we will calculate the azimuthal temperature $\bar{T}_{k\theta}$ averaged over the plasma column cross section.

$$\bar{T}_{k\theta} = T_k (\text{when } L_k \gg R_k^*), \quad \bar{T}_{k\theta} = \left(\frac{T_k}{4\pi} \frac{L_k}{R_k^*}\right)^2 (\text{when } L_k \ll R_k^*).$$

Thus, if the longitudinal magnetic field is strong, then the averaged azimuthal temperature equals the radial. If the magnetic field is small then the azimuthal temperature is significantly less than the radial. Using Expressions (11), (14)-(18), we will calculate the azimuthal current I_k which refers to a unit of column length, and calculate the azimuthal velocity $\bar{V}_k$ averaged over the column section.

$$\begin{gathered} I_k = \sqrt{2}\,\pi^{1/4} Q_k e_k u_k \frac{R_k^*}{L_k} \operatorname{sign} \omega_k, \quad \bar{V}_k = \sqrt{2}\,\pi^{1/4} u_k \frac{R_k^*}{L_k} \operatorname{sign} \omega_k \ (L_k \gg R_k^*), \\ I_k = \sqrt{\frac{2}{\pi}}\, Q_k e_k u_k \operatorname{sign} \omega_k, \quad \bar{V}_k = \sqrt{\frac{2}{\pi}}\, u_k \operatorname{sign} \omega_k \ (L_k \ll R_k^*). \end{gathered} \tag{19}$$

Consequently, if the magnetic field is small then the averaged ordered velocity is close to the heat velocity, and the current is proportional to the product of the number of particles times the heat velocity. With strong magnetic fields, the velocities of ordered movement are small compared with heat velocities (the magnetic field "freezes" the degrees of freedom) and the azimuthal current is found to be inversely proportional to the magnetic field strength.

7. We will set the temperatures and space currents of all types of ions as equal to zero, and we will consider the space current at each point as compensated. Then the electron distribution will be determined by the magnetic field, temperature, etc. In connection with this, the first and third equations of System (4) are satisfied identically, and the second in the case of the distribution function of type (10) takes the form

$$\frac{d^2 B}{d\eta^2} - \Omega^2 \frac{B}{\eta} e^{-\frac{B^2}{\eta}} = 0, \quad \Omega^2 = \frac{\pi e^2 n_0 r_0^2}{c^2 m}, \quad \eta = 1 + \left(\frac{r}{r_0}\right)^2, \quad B = \frac{eAr}{cr_0\sqrt{2m\varkappa T}}. \qquad (20)$$

If the azimuthal component of the vector potential vanishes at the axis of symmetry, then close to the column axis (when $(\frac{r}{r_0})^2 \ll 1$) the approximate solution of the equation in (20) has the form

$$B = \frac{r^2 cH}{2cr_0\sqrt{2m\varkappa T}} \left\{1 + \frac{\Omega^2}{2}\left(\frac{r}{r_0}\right)^4 \left[\frac{1}{3} - \frac{1}{4}\left(\frac{r}{r_0}\right)^2 + \frac{1}{4}\left(\frac{r}{r_0}\right)^4 \left(\frac{3}{4} - \frac{e^2 r_0^2 H_0^2}{2\varkappa c^2 m\varkappa T} - \frac{\Omega^2}{15}\right)\right]\right\} + \dots \qquad (21)$$

With the aid of (16), (17) we carry out the analysis of Solution (21) in which the first term corresponds to a uniform magnetic field. If the thickness of the plasma layer is less than the Larmor radius ($L \ll R^*$), then the additional terms are small compared to the first term with the fulfillment of the inequalities

$$\frac{r}{L} < \frac{L}{2\pi R^*}, \quad \left[\frac{\left(\frac{H}{4\pi}\right)^2}{n_0 \varkappa T}\right]^{1/4}.$$

If the Larmor radius is small compared with the thickness of the plasma layer ($L \gg R^*$), then the additional terms are small under the following conditions

$$\frac{r}{L} < 1, \quad \left[\frac{\left(\frac{H}{2\pi}\right)^2}{n_0 \varkappa T}\right]^{1/4}.$$

Thus, the approximation is found to be satisfactory only inside the plasma layer. The limits in which this approximation is valid are wider, the stronger the magnetic field, the lower the temperature and the less the density. The solution in (21) takes into account, in the form of terms of a higher order with respect to radius, the nonlinear effects associated with the influence of characteristic plasma currents on the magnetic field and distribution of density. With sufficiently large radii, the magnetic field is uniform and solutions examined in a previous paragraph are satisfactory.

8. Let the temperatures of all types of ions be small compared with the electron temperature. In the case of small density ($n \to 0$, $\varphi = 0$, $A = \frac{Hr}{2}$), using relations (11), (12), (14)-(17), we will obtain the expressions for the average value of the azimuthal component of velocity V, azimuthal temperature $T_\theta(r)$ and electron density $n(r)$;

$$V = \frac{2\pi\delta\omega R_* r^3}{L^4 + 4\pi\delta R_*^2 r^2}, \quad T_\theta = \frac{T L^4}{L^4 + 4\pi\delta R_*^2 r^2}, \quad \delta = 2^{-\operatorname{sign}(R_* - L)},$$
$$n(r) = \frac{n_0 L^2}{\sqrt{L^4 + 4\pi\delta R_*^2 r^2}} \exp\left[-\frac{\pi\delta r^4}{2(L^4 + 4\pi\delta R_*^2 r^2)}\right]. \qquad (22)$$

If the Larmor radius of electrons is large compared with the thickness of the plasma layer, then when $r \sim L$ the density is significantly

less than at the axis. When $L \leqslant r \leqslant R_*$ the density of plasma is small, but the number of particles in this layer is large. At a sufficiently large distance from the axis of symmetry the number of particles is very small.

If the Larmor radius is small compared with the thickness of the plasma layer, then from (22) it follows that close to the axis the density is almost constant. When $r \sim L$ the density sharply decreases so that at great distances from the axis there are few particles. Consequently with a fixed electron temperature the column has a sharp boundary at sufficiently strong magnetic fields.

9. In the study distribution functions depending only on integrals of symmetry $\mathcal{H}$, P_2, P_3 are examined. Not diminishing the generality, the distribution function can be taken as exponents

$$f = f_0 \exp S(\mathcal{H}, P_2, P_3).$$

The analogy of entropy S artibrarily depends on the integrals of symmetry. In the general case, in a sufficiently small neighborhood of any nonsingular point ($\mathcal{H}_0$, P_{20}, P_{30}) the value S can be presented as a Taylor series

$$S = \sum_{kmn=0}^{\infty} a_{kmn} (\mathcal{H} - \mathcal{H}_0)^k (P_2 - P_{20})^m (P_3 - P_{30})^n.$$

If the physical conditions of the problem are such that the fundamental contribution to the value S is made by a terms of a lower order [in the general case – in a sufficiently small neighborhood of a nonsingular point ($\mathcal{H}_0$, P_{20}, P_{30})], then, having eliminated terms of higher order, we will obtain for S an expression in the form of a certain polynomial with respect to the Hamiltonian and momenta. In the cases examined in this study, the polynomial linearly depends on the Hamiltonian and is a linear or quadratic

function of momenta. With respect to its formal indication such a distribution function is close to Maxwellian.

In the case of linear dependence of S on momenta, the temperature is constant, uniform and isotropic. Generalized velocities are constant, components of linear velocity are proportional to corresponding Lame's coefficients and do not depend on electrical or magnetic potentials. The ordered movement of each of the types of ions occurs as a movement of a solid. In a Cartesian system of coordinates this case corresponds to interpenetrable plasma globs whose ordered velocities are constant, and in a cylindrical system it corresponds to plasma globs moving along the axis of symmetry with constant velocities and rotating with constant angular velocities

In the case of a quadratic dependence on momenta, the temperature, generally speaking, is anisotropic and evidently does not depend on electrical and magnetic potentials. In a Cartesian system of coordinates thr temperature is constant and uniform and in a curvilinear system depends on Lame's coefficients. Constants, characterizing the plasma state, enter into the expressions for temperature and velocity. The velocities are linear functions of magnetic potential and depend on Lame's coefficients.

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